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EFFECT OF FINITE LENGTH ON FORCED CONVECTION HEAT TRANSFER FROM CYLINDERS

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Abstract – An experimental investigation was undertaken of the effect of aspect ratio, that is to say, length to diameter ratio, on forced convection heat transfer from single cylinders of finite length in cross flow. A correlation of the data was obtained of the form

$$Nu = cRe^n + b\left(\frac{D}{L}\right)^l Re^m.$$

It appears that there is little effect of aspect ratio for values greater than four.

The disagreement between the correlations proposed by Zukauskas [1] and by Morgan [2] for large aspect ratio, based on several hundred earlier investigations, was considered and resolved.

The above correlation agrees well with these works for large aspect ratio and agrees also with the well known correlation for isothermal flat plates in the limit of small aspect ratio.

NOMENCLATURE

a,	aspect ratio of cylinder L/D;
b, c,	coefficients in correlation;
C_p,	specific heat at constant pressure;
D,	diameter of cylinder;
h,	film heat-transfer coefficient;
K,	conductivity;
l, m, n,	indices in correlation;
L,	length of cylinder;
Nu,	Nusselt number, hD/K;
Pr,	Prandtl number $\mu C_p/K$;
q,	heat flux density;
Re,	Reynolds number UD/ν;
T,	absolute temperature;
Tu,	turbulence intensity u/U;
U,	velocity of fluid stream;
u	root mean square velocity fluctuation in direction of flow;
μ,	absolute viscosity;
ν,	kinematic viscosity.

Subscripts

c,	cylinder surface value;
f,	film value;
s,	stream value.

INTRODUCTION

THERE have been very many studies both theoretical and experimental of forced convection heat transfer from the surface of heated cylinders in cross flow. Although the length to diameter ratio is a very relevant parameter it has received rather less attention than its importance merits. In many practical designs it is assumed that data for very long thin cylinders, that is, equivalently infinite aspect ratio, is applicable to cases of small aspect ratio. There has been no systematic investigation of the effect of aspect ratio as such. Indeed, accurate correlation of the available data was not forthcoming even for the special case of long cylinders. The separate effects of several relevant parameters were not properly quantified.

Recently, however, Zukauskas [1] and Morgan [2] have written comprehensive review articles covering several hundred researches. That of Morgan is the more extensive of the two and he systematically identifies the different relevant parameters.

In the case of turbulent forced convection the following are important: solid and wake blockages in the wind tunnel; turbulence level; and temperature loading leading to significant variation in fluid properties.

As Morgan points out failure to take the effect of these parameters properly into account has lead to disparate results in attempts to produce overall correlations from the very many sets of experimental data. A re-assessment of results for smooth cylinders lead Morgan to propose a correlation of the form

$$Nu_f = cRe_f^n \tag{1}$$

for zero turbulence, low temperature loading conditions which, he claimed, fitted existing data to $\simeq 5\%$ over a Reynolds number range from 10^{-4} to 2×10^5. The coefficient c and index n have each seven different values according to the part of the Reynolds number range and these are given in Table 1. Careful corrections were applied to the available data for the effects of the parameters given above and Nusselt and Reynolds number re-calculated on a basis of film temperature, $T_f = \frac{1}{2}(T_c + T_s)$, using modern accurate values for viscosity and conductivity.

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Table 1. Coefficients in the overall correlations of Zukauskas [1] and Morgan [2]

Zukauskas [1]				Morgan [2]			
Re_s				Re_f			
from	to	c	n	from	to	c	n
1	40	0.658	0.4	10^{-4}	4×10^{-3}	0.437	0.0895
40	1×10^{3}	0.449	0.5	4×10^{-3}	9×10^{-2}	0.565	0.136
1×10^{3}	2×10^{5}	0.229	0.6	9×10^{-2}	1	0.800	0.280
2×10^{5}	1×10^{6}	0.0666	0.7	1	35	0.795	0.384
Zukauskas [1] modified				35	5×10^{3}	0.583	0.471
Re_f		c	n	5×10^{3}	5×10^{4}	0.148	0.633
				5×10^{4}	2×10^{5}	0.0208	0.814
1	40	0.645	0.4	Present results			
40	1×10^{3}	0.447	0.5	Re_f		c	n
1×10^{3}	2×10^{5}	0.231	0.6				
2×10^{5}	1×10^{6}	0.0683	0.7	6×10^{4}	2.2×10^{5}	0.123	0.651

Zukauskas [1] proposed correlations for a higher range of Reynolds number $1 < Re < 10^6$. In addition to the effect of turbulence, Zukauskas was concerned to evaluate the effect of the Prandtl number of the fluid and of the thermal boundary conditions. The latter should include considerations not only of whether the cylinder or the fluid is heated, but also of whether the heated cylinder surface is at constant temperature or subject to uniform heat flux. Zukauskas gave the correlation

$$Nu_s = cRe_s^n Pr_s^{0.37}\left(\frac{Pr_s}{Pr_c}\right)^{0.25} \tag{2}$$

covering a Prandtl number range up to about 10 and being valid for both heated and cooled cylinders with moderate temperature loading. This correlation is based on evaluating the fluid properties at stream temperature and accordingly is not directly comparable with that of Morgan. However, for gases at moderate temperatures, $T_c/T_s < 1.20$ for heated cylinders, it simplifies to equation (1) also and the coefficients can be adjusted to take account of evaluating fluid properties at film temperature. Table 1 gives the Zukauskas values for c and n, based on T_s, together with recalculated values based on T_f. There is little difference.

The correlations of Morgan and Zukauskas are compared in Fig. 1, for $Re > 40$, where it can be seen that there is good agreement between them over most of the range. They differ no more than 7% from their average value for $Re < 10^4$. However for $Re = 2\times10^5$ the difference is 11% with Morgan's correlation predicting a value for Nu which is 27% higher than that of Zukauskas. Also if we extend Morgan's correlation to the upper limit of the Zukauskas correlation the difference at $Re = 10^6$, is 51%.

Apart from considering which of the two is more correct the question arises whether either has properly taken into account the effect of aspect ratio. Clearly an equation of the form of equation (1) which uses only Nu and Re as the relevant dimensionless groups is correct only for cylinders of infinite length. That is for correlating data obtained from cylinders of very large aspect ratio, $a > 100$ say, or data obtained using a 'guard-ring' technique.

Further, as Zukauskas pointed out, a correct evaluation of the film coefficient, and thence Nu, depends on an accurate determination of the local heat flux density and temperature at every point on the cylinder surface. This can be done either by calculating the local value of h everywhere and integrating to get a mean value or by dividing the mean heat flux by the mean temperature difference. The resulting values for h_m are identical for an isothermal surface but a considerably higher value results from the first method if the surface heat flux is constant. In evaluating experimental data the second method is used mostly and gives approximately the same value whether the experimental conditions are of constant surface temperature or of constant heat flux.

Although Morgan took proper account of the effect of aspect ratio and temperature measurement in proposing a correlation for natural convection it is clear that in many of the studies quoted by him and used as a basis for his correlation for forced convection this was not so. The same is true about the Zukauskas correlation. In many of the studies used the aspect ratio was small, $a < 10$, without the use of a guard-ring, and in some cases the actual experimental technique, large cylinders heated with condensing steam or small fine wires heated electrically, may not have lead to an accurate determination of the film coefficient. However, the good agreement between the two correlations establishes their correctness with little doubt; at least over the range in which they do agree well, that is, $Re < 5\times10^4$.

Accordingly it seemed that a further experimental study of this problem would be of value. Its main aim should be to determine the effect of aspect ratio. In addition, for infinite cylinders, with regard to the difference between the correlations proposed by Zukauskas and Morgan the study should cover the

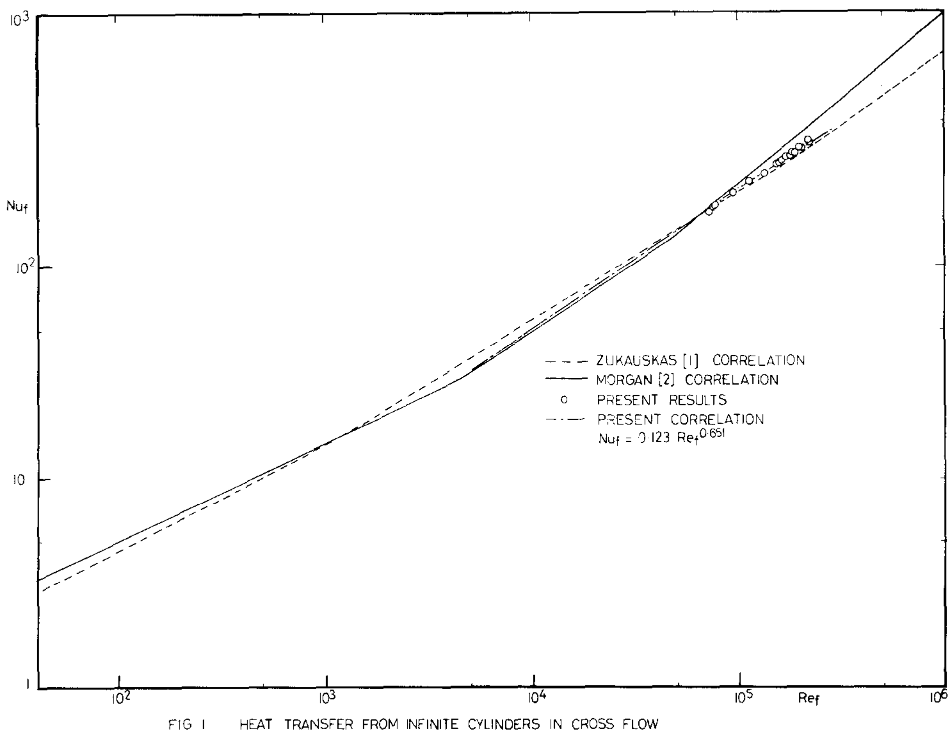

FIG. 1. Heat transfer from infinite cylinders in cross flow.

Reynolds number range over which they disagree and part of the range over which they agree. The study should be for low turbulence conditions and low temperature loading and all necessary corrections should be applied.

EXPERIMENTAL INVESTIGATION

(a) *Apparatus*

Clearly a good measurement of the film coefficient would be best achieved by using a heated cylinder whose surface temperature is accurately determined to be constant over the whole of the surface. In addition the cylinder should be so constructed as to enable the effect of aspect ratio to be determined as well as making it possible for the guard-ring technique to be used.

Accordingly the heated cylinder was machined from a solid billet of aluminium and consisted of five sections carefully spigotted together each with a separately controllable heater mounted on a central ceramic core. The wall thickness was considerable, 12 mm, so that circumferential variations of surface temperature were negligible. Surface temperatures were measured by thermocouples, 19 in number, arranged so that axial and circumferential differences could be determined. The overall size of the cylinder was 750 mm long by 150 mm dia. The arrangement is shown in Fig. 2. It was mounted horizontally in a low turbulence wind tunnel of 0.92 mm square section and supported by fine wires.

Clearly the use of individually controlled heaters allows the guard ring technique to be employed and allows the effect of aspect ratio to be determined also. Thus, by careful adjustment of the heaters it was possible to ensure a uniform surface temperature and, since the heaters were individually controlled, the mean surface heat flux of each section could be deduced. Each section was adequately thermally isolated by use of thick asbestos rings and fibreglass wool to prevent any heat transfer in the core by conduction, radiation or natural convection. In a steady state heat generated by the heater coil of each section passes out through the cylinder surface of that section.

(b) *Data collection*

The various cylinder surface and fluid stream temperatures were taken for fluid speeds up to 21 m/s. The fluid velocity was corrected for blockage effects using the formula proposed by Morgan but no correction was necessary for turbulence level in this case as $Tu < 0.001$.

The surface temperature of the cylinder was about 50°C and the ambient fluid temperature about 25°C giving a temperature loading factor $T_c/T_s = 1.08$ and a film temperature T_f about 310 K. By careful adjustment of the heaters it was possible to keep the surface temperature, T_c, constant over the surface within ± 0.5°C that is $\pm 0.3\%$. These slight variations were averaged to determine the film coefficient. In determining the surface heat flux for each section corrections were applied for axial conduction and radiation. Both were slight.

The effect of aspect ratio was investigated as follows.

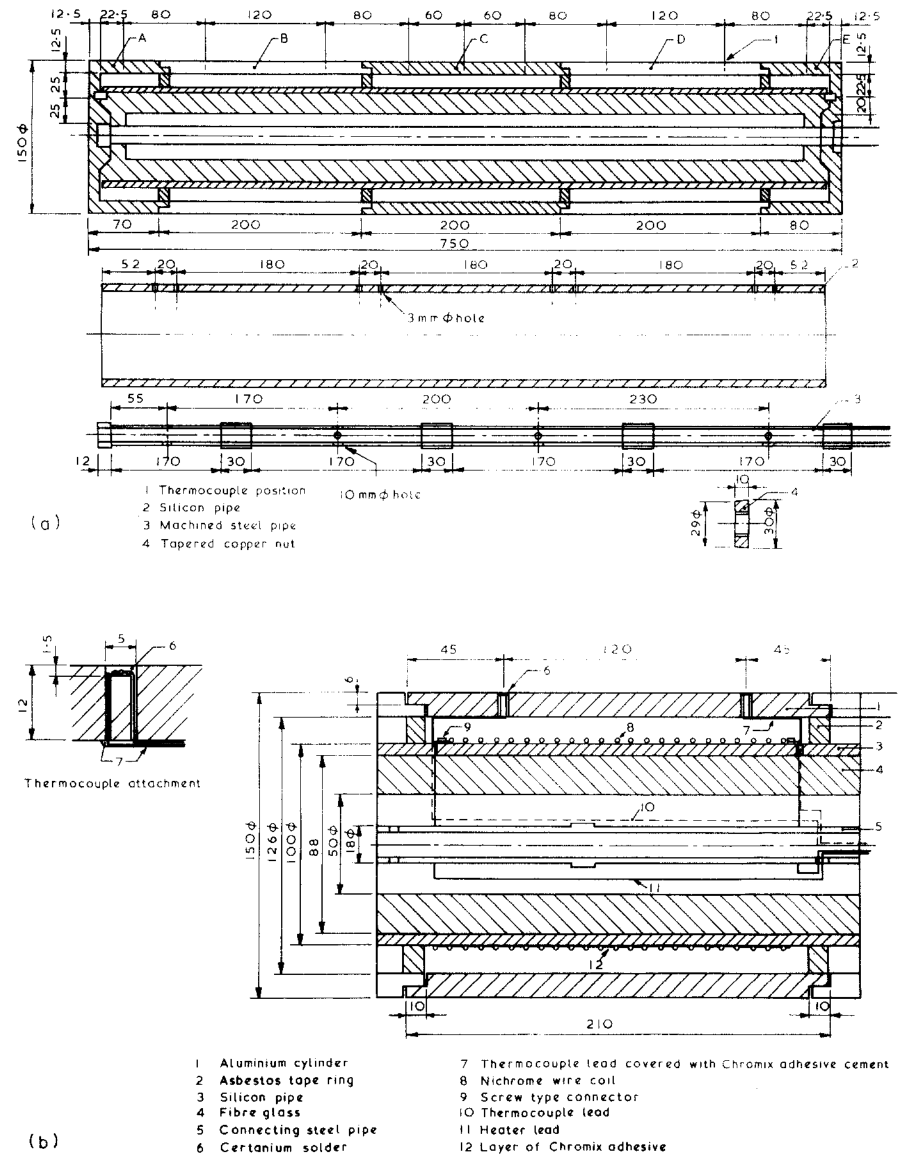

FIG. 2. Section of heated cylinder. (a) Test cylinder construction detail; (b) section heating system, thermocouple attachment and insulation details.

Taking the centre section (C) only, we have in essence an application of the guard ring technique so that heat transfer is equivalently as from a cylinder of infinite aspect ratio. This is true also for the composite section comprised by adding together the centre section with either or both of the sections which lie on each side of it, B and D, Fig. 2.

On the other hand a section, B or D, adjacent to an end section, A or E, can be regarded as a guard ring section for a cylinder of small aspect ratio. Similar considerations apply to the other possible combinations so that six different values of a could be investigated, namely, a equal to 1, 3.66, 5, 6.33, 9.0 and ∞.

Full details of all the measurements and corrections for the present investigation are given by Al-Fakhri [3].

RESULTS AND DISCUSSION

The results for infinite cylinders with constant surface temperature for low turbulence and low temperature ratios are shown in Fig. 1, where they are compared with the correlations of Zukauskas and of Morgan. They agree well with both correlations for $7 \times 10^4 < Re < 1.1 \times 10^5$ and when extrapolated agree well with Morgan's correlation down to $Re = 6 \times 10^3$. For the range in which Zukauskas and Morgan disagree the present experimental results agree best with Zukauskas. They are correlated by:

$$Nu_f = 0.123\, Re_f^{0.651}. \tag{3}$$

This agrees well as regards power index, $n = 0.651$, with the value given by Zukauskas, $n = 0.6$, for this range of Reynolds number but disagrees rather markedly with the value given by Morgan, $n = 0.814$.

On examining the many researches surveyed by Morgan it may be noticed that only one, that of Hilpert [4] suggests a value for n of 0.8 whilst several dozen suggest values between 0.6 and 0.7. Morgan laid particular emphasis on the value of Hilpert's work since the experimental conditions were of virtually zero blockage and very low turbulence and low temperature loading. However in the experimental set-up used by Hilpert it is not at all certain that the surface temperature was constant or accurately determined. We consider that Morgan over-emphasized the importance of the Hilpert data for the higher range of Reynolds number.

The effect of aspect ratio is shown in Fig. 3 where it appears that for $a > 4$ there is little further effect as a increases to infinity. This contrasts with the finding of Morgan for natural convection who noted an effect of aspect ratio for values up to $a = 10^4$. Much of the data on which this finding was based was from fine wires held between thick supports as is the case in a hot wire anemometer. It is probable that the apparent effect of aspect ratio was in fact an effect of conduction to the supports. These of course have very much larger conducting surfaces than the fine wires they support whereas in the present work the opposite is the case.

However, it is clear from the present results that Morgan and Zukauskas were correct to incorporate data from cylinders of finite aspect ratio in their correlations for forced convection.

It was found possible to correlate the effect of aspect ratio, thus:

$$Nu_f = 0.123 Re_f^{0.651} + 0.00416\left(\frac{D}{L}\right)^{0.85} Re_f^{0.792}, \quad (4)$$

which of course reduces to the above correlation, equation (3), for infinite cylinders when the aspect ratio is large.

On the other hand when the aspect ratio is small the result should be comparable with correlations proposed for heat transfer from isothermal flat plates because for small aspect ratio, little of the heat transfer is from the curved surface of the cylinder and nearly all of it from the flat ends. In the limit the first term of equation (4) becomes negligible compared with the second term; the Reynolds number index of which, $m = 0.792$, compares very well with the value, $m = 0.8$, in the well known correlation for turbulent flow over isothermal flat plates.

In establishing the value of the results for small aspect ratio it is necessary to consider whether the flow pattern near the end of the cylinder properly simulates the flow pattern of a small aspect ratio cylinder similarly placed in a uniform stream. A fundamental assumption of the work of Morgan and Zukauskas and the many others who have sought experimental correlations of the form of equation (1) is that the flow pattern is uniquely determined by the Reynolds number and accordingly we do not have to consider the exact detail of the flow pattern. So it is sufficient for the present purpose to establish that there was no significant axial variation of velocity at the plane which is equivalently the central plane of the cylinder of smallest aspect ratios; that is, at the plane separating sections A and B. Velocity measurements showed this to be the case.

In addition, we can regard sections B and C taken together as an equivalent infinite cylinder, the guard rings in this case being sections A and D respectively. Clearly any significant axial velocity variation across the plane separating sections A and B would have an effect so that the results for sections B and C taken together would differ from the result for section C alone. It was found that the average surface temperature and heat flux taken over B and C together differed less than 1% from the values taken from C alone for the

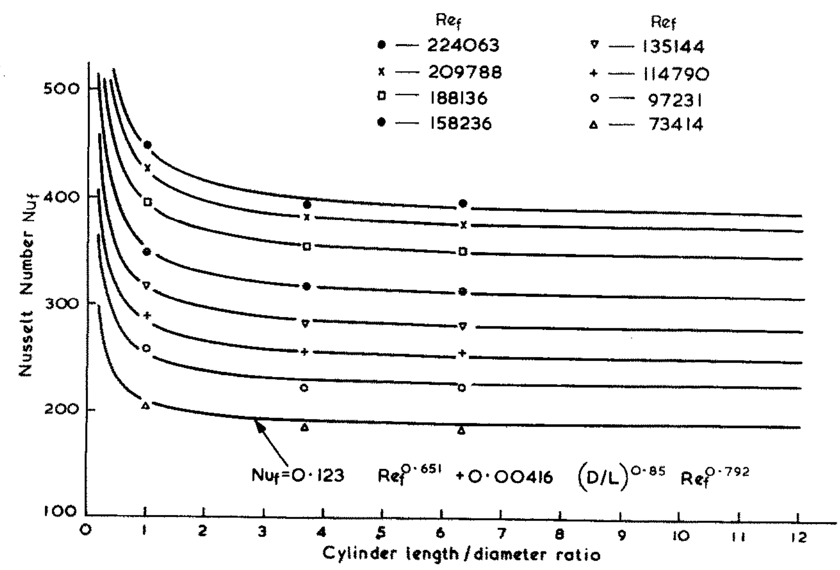

FIG. 3. Effect of aspect ratio on heat transfer from cylinders.

same Reynolds number and the corresponding Nusselt numbers were indistinguishable. Clearly there was no significant effect of axial velocity at the plane separating sections A and B on the heat-transfer correlation for infinite aspect ratio and consequently neither could there be any on the correlation for small aspect ratio using section A alone. It is concluded that the flow pattern for small aspect ratios was properly simulated.

However, when section D was taken with B and C, a small but detectable difference of 5–7% was noted in the resulting Nusselt numbers. This is less than the difference between the Morgan and Zukauskas correlations but it is concluded that it was significant and was caused by the presence of the heating current wires which entered the cylinder through the end face of section E. This suggests that the present set-up simulates the required flow pattern rather better than might be the case with a short heated cylinder to one end of which heating wires and supports were attached.

CONCLUSION

The correlations proposed by Zukauskas [1] and Morgan [2] based on data from several hundred experimental works were found to disagree significantly for $Re > 7 \times 10^4$. New experimental evidence was obtained which resolves the difference in favour of the Zukauskas correlation. In acquiring this new data care was taken in the design of the apparatus and in evaluating the results to avoid the several sources of error recognized by these two authors as leading to rather wide disparities between earlier studies.

For heat transfer to or from a smooth cylinder in a forced convection cross flow with low turbulence and low temperature loading the correlations proposed by Morgan or Zukauskas are correct for $Re < 7 \times 10^4$ whilst for $7 \times 10^4 < Re < 10^6$ the correlation of Zukauskas is correct.

The present results indicate that these correlations are correct not only for very long thin cylinders but also for cylinders with an aspect ratio as low as about four.

A correlation is proposed, equation (4), for heat transfer from cylinders of low aspect ratio which in the limit agrees with the above correlation for large aspect ratio and with the generally accepted correlation for turbulent heat transfer from isothermal flat plates for small aspect ratio.

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EFFET DE L'ALLONGEMENT DES CYLINDRES SUR LE TRANSFERT THERMIQUE PAR CONVECTION FORCEE

Résumé — On étudie expérimentalement l'effet du rapport d'allongement, c'est à dire le rapport longueur sur diamètre, sur la convection thermique forcée d'un cylindre de longueur finite, en attaque transversale. On obtient la formule

$$Nu = cRe^n + b\left(\frac{D}{L}\right)^l Re^m.$$

L'effet du rapport d'allongement est faible pour des valeurs supérieures à quatre. Le désaccord entre les formules proposées par Zukauskas [1] et par Morgan [2] pour les grands rapports, est considéré et résolu à partir de plusieurs centaines d'études antérieures.

La formule proposée s'accorde bien avec ces travaux pour de grands rapports d'allongement, et aussi avec la formule connue des plaques planes isothermes, dans la limite des faibles rapports.

EINFLUß DER ENDLICHEN LÄNGE AUF DEN WÄRMEÜBERGANG DURCH ERZWUNGENE KONVEKTION AN ZYLINDERN

Zusammenfassung—Der Einfluß des Längenverhältnisses, d. h. des Verhältnisses von Länge zu Durchmesser, auf den Wärmeübergang durch erzwungene Konvektion bei einzelnen querangeströmten Zylindern endlicher Länge wurde experimentell untersucht. Die Daten wurden in folgender Form korreliert

$$Nu = cRe^n + b\left(\frac{D}{L}\right)^l Re^m.$$

Es hat den Anschein, daß der Einfluß bei Längenverhältnissen größer als vier gering ist.

Der Widerspruch zwischen den Korrelationen von Zukauskas [1] und Morgan [2] für große Längenverhältnisse, die auf einigen hundert früheren Untersuchungen aufbauen, wurde in Betracht gezogen und geklärt. Die obige Korrelation stimmt für große Längenverhältnisse gut mit diesen Arbeiten überein, und sie stimmt ebenso für kleine Längenverhältnisse mit der wohlbekannten Korrelation für ebene, isotherme Platten überein.

ВЛИЯНИЕ КОНЕЧНОСТИ ДЛИНЫ ЦИЛИНДРОВ НА ТЕПЛООБМЕН ПРИ ВЫНУЖДЕННОЙ КОНВЕКЦИИ

Аннотация — Проведено экспериментальное исследование влияния отношения длины одиночного поперечно обтекаемого цилиндра к диаметру на перенос тепла при вынужденной конвекции. Получено следующее соотношение

$$Nu = cRe^n + b\left(\frac{D}{L}\right)^2 Re^m.$$

При отношении длины цилиндра к диаметру, превышающем 4, обнаружено слабое влияние на теплоперенос. Дано объяснение расхождению между корреляциями Жукаускаса и Моргана при больших отношениях длины к диаметру. Предложенная зависимость для числа Нуссельта обобщает результаты этих авторов при больших отношениях длины к диаметру, а также согласуется с известным соотношением для изотермических плоских пластин для предельно малых величин указанного отношения.